Image Deformation using Radial Basis Function Interpolation

Jung Hye Kwon, Byung Gook Lee, Jungho Yoon, Jun Jae Lee

kjh@dit.dongseo.ac.kr, lbg@dongseo.ac.kr,
yoon@ewha.ac.kr, joonlee@kmu.ac.kr

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Image Deformation

- As one field of computer graphics
- The deformation method of changing image to be wanted by user
  - Used in the filed of computer animation, morphing and medical image
- To perform deformation the user selects some set of handle
  - Points, lines, or grids

Previous Deformation Techniques

- Mesh base method
  - T. Igarashi, T. Moscovich, and J. F. Hughes, "As-rigid-as-possible shape manipulation .", ACM Trans. Graph 2005, 24, 3, pp 1134-1141 (2005).
  - Y. Weng, W. Xu, Y. Wu, K. Zhou, B. Guo, "2D shape deformation using nonlinear least squares Optimization.", The visual computer, pp 653-660(2006).
- Approximation method
  - S. Schaefer, T. McPhail, J. Warren, "Image deformation using moving least squares.", Proceedings of ACM SIGGRAPH , pp. 533-540 (2006).
  - N. Arad, N. Dyn, D.Reisfeld, Y.Yeshurun, "Image warping by radial basis functions :Application to facial expressions", Computer Vision Graphics and Image Processing , p.p 161-172 (1994).

Radial Basis Function

- A function $f:\mathbb{R}^{d_1}\to\mathbb{R}^{d_2}$ is known only at a set of discrete points $U:=\{u_1,u_2,\cdots,u_n\}$ and desired function values $V:=\{v_1,v_2,\cdots,v_n\}$, we can define

$$S_{f,U}(u)=\sum_{i=1}^{n}\alpha_i\phi(\|u-u_i\|)+\sum_{j=1}^{m}\beta_j p_j(u)$$

with the constraints

$$\sum_{i=1}^{n}\alpha_i p_j(u_i)=0,\ j=1,2,\cdots,m$$

Here, $p_j(u)\in\Pi_r^{d_1}$ is the space of polynomial of total degree r in d spatial dimensions

- Calculate the coefficients of $S_{f,U}(u)$ that are acquired to satisfy $(n+m)\times(n+m)$ system of linear equations. We may be written in matrix form as

$$\begin{pmatrix} A & P \\ P^T & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}=\begin{pmatrix} V \\ 0 \end{pmatrix}$$

- Unique solution is obtained in case of the inverse of matrix

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix}=\begin{pmatrix} A & P \\ P^T & 0 \end{pmatrix}^{-1}\begin{pmatrix} V \\ 0 \end{pmatrix}$$

where

$$A=\begin{pmatrix} \phi(\|u_1-u_1\|) & \phi(\|u_1-u_2\|) & \dots & \phi(\|u_1-u_n\|) \\ \phi(\|u_2-u_1\|) & \phi(\|u_2-u_2\|) & \dots & \phi(\|u_2-u_n\|) \\ \vdots & \vdots & \ddots & \vdots \\ \phi(\|u_n-u_1\|) & \phi(\|u_n-u_2\|) & \dots & \phi(\|u_n-u_n\|) \end{pmatrix},\quad P=\begin{pmatrix} p_1(u_1) & p_2(u_1) & \dots & p_m(u_1) \\ p_1(u_2) & p_2(u_2) & \dots & p_m(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ p_m(u_n) & p_m(u_n) & \dots & p_m(u_n) \end{pmatrix}$$

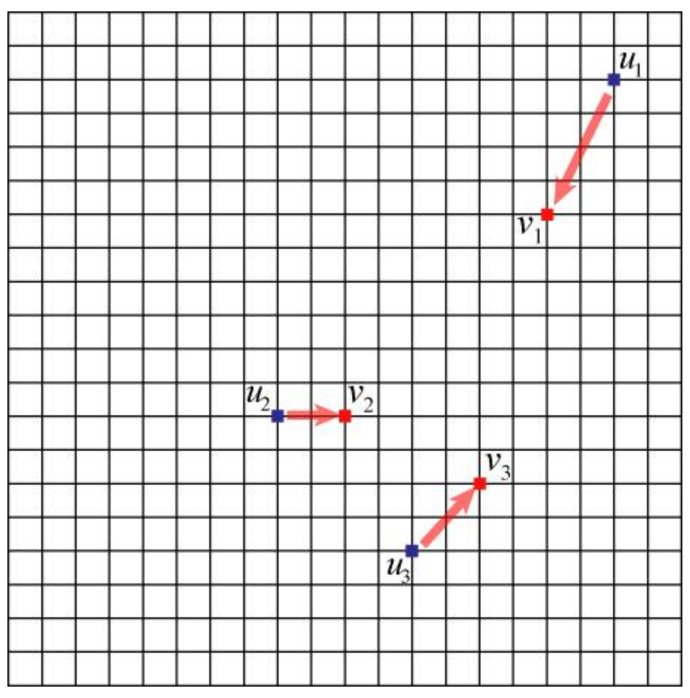
Image deformation using RBF

- Since constructing a deformed image from an original one is a mapping from $\mathbb{R}^2$ to $\mathbb{R}^2$, we have given two sets of 2-dimensional data $U:=\{u_1,u_2,\cdots,u_n\}$ and deformed position $V:=\{v_1,v_2,\cdots,v_n\}$. We solve for the radial basis function interpolation $S_{f,U}(u)$, satisfying

$$S_{f,U}(u_i)=v_i-u_i,\quad i=1,2,\cdots,n.$$

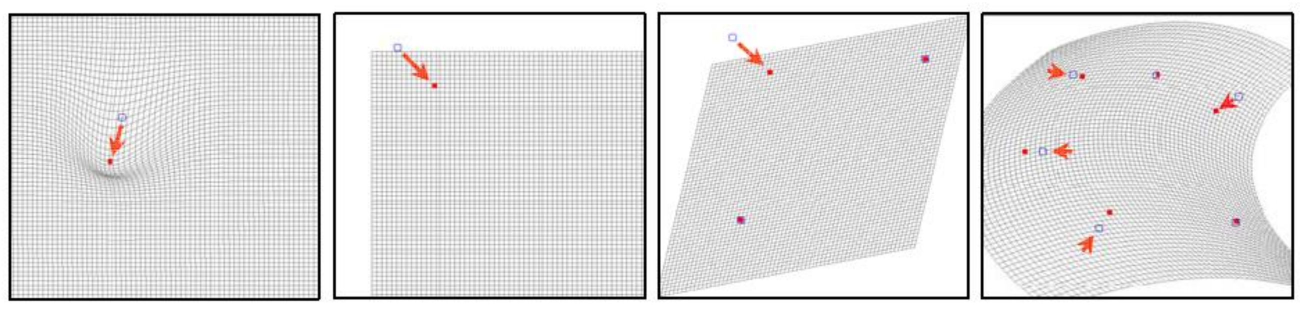
where v_i-u_i is difference vector.
Finally, we obtain a deformed position v

$$v=u+S_{f,U}(u)$$



Experimental Result

- The deformation result according to polynomial degree

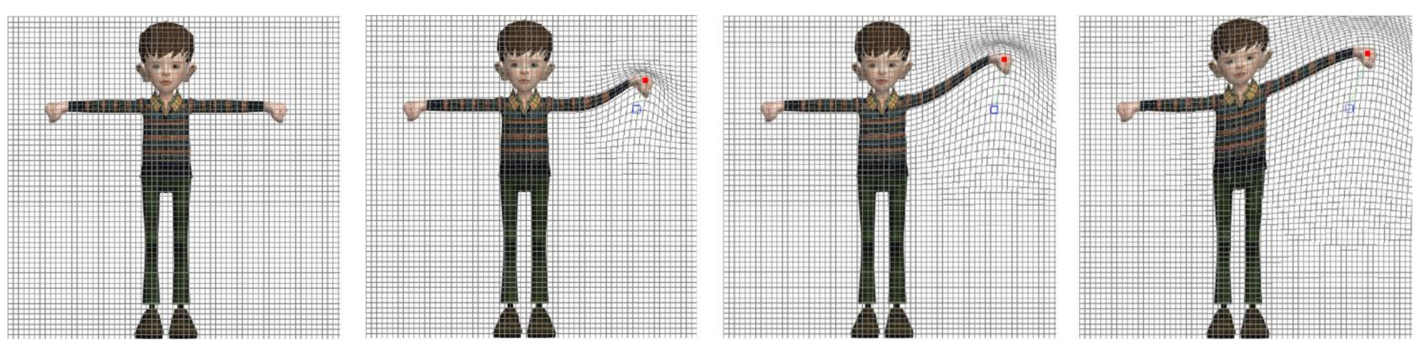


(a) without polynomial (b) with constant (c) with linear (d) with quadratic

- Test using Gaussian function $\phi(r)=e^{-r^2/c^2}$ and Wendland's function $\phi_{3,1}(r)=(1-\frac{r}{c})_+^4+(4\frac{r}{c}+1)$ $c=10$

(a) Original image (b) Gaussin function (c) Wendland's function

- An illustration of RBF interpolation with different values of c

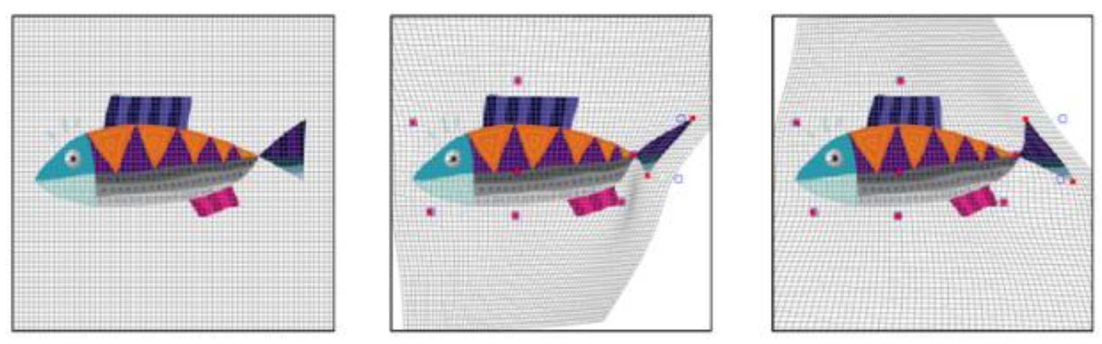


(a) Original image (b) $c=5$ (c) $c=10$ (d) $c=15$

- Comparison between our algorithm and [Schaefer et al.]

(a) Original image (b) Our algorithm (c) Schaefer et al.

- Deformation of a fish image



RBF with quadratic

Conclusion & Future work

- Our proposed method is faster by simple calculation and its result is better than the previous methods.
- The term of controlling polynomial degree has many possibilities for various application fields.
  - Especially, in field of animation.
- Further research will be extended to uses of other basis functions.
- line or curve segments instead of points as control attribute.