Analysis of the equilateral grid-based template and automatic triangular initial mesh generation

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2D mesh generation methods

Classified by Ho-Le(*Computer-Aided Design*, 1988; **20**; 27-38.) and summarized in (A. Hu and M. F. Randolph, *Int. J. Num. Meth. Eng.*, 1998; **22**; 327-250.) as the following approaches :

- Nodal connecting approach
- Grid-based approach
- Mapped element approach
- Decomposition approach.

Grid-based approach

An algorithm for a grid-based approach proposed by Thacker *et al.*(see *Journal of Computational Physics*, 1980; **37**; 371-387) is considered to generate the initial mesh.

General grid-based algorithm follows three steps :

1. The grid is superimposed on the object.

2. The grid cells that fall outside the object are removed.

3. The grid cells that intersect the object boundary are adjusted or trimmed so that they fit into the object.

Advantage and Requirements

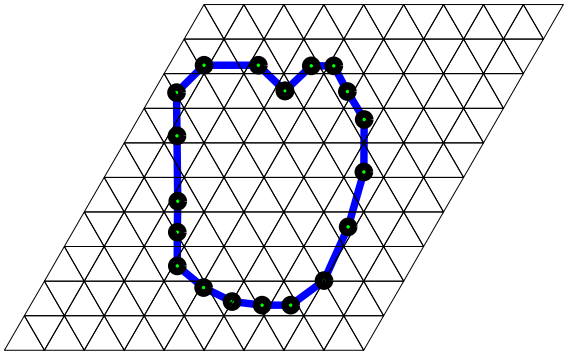
An advantage of this method is that the constraint of nearly equilateral elements can be satisfied because construction begins with exactly equilateral elements, and neighboring points can be systematically identified because the initial grid is regular, and it could be implemented in most finite element software.

Then there are several requirements. The Step 2 requires to recognize which points are exterior and which are interior because the exterior points are excluded, in order to obtain the zigzag grid. Also, it needs the standard of choosing the zigzag boundary and the method for reindexing the interior and boundary points. In the Step 3, the method for adjusting the zigzag boundary to the boundary of the object has to be considered.

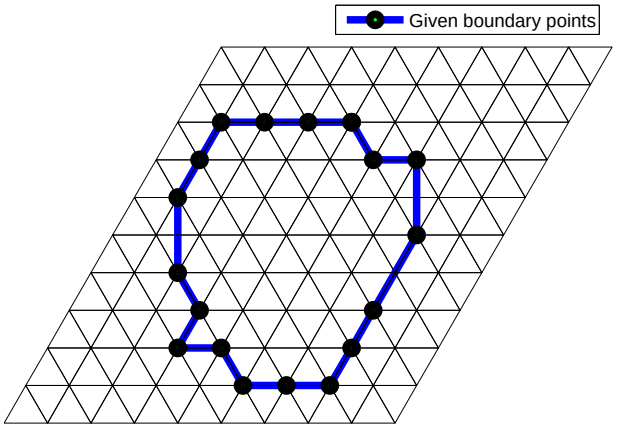
Researchers

- Thacker *et al.* (1980)
- K. K. Wang, N. Hashimoto : A geometric modelling system (1981)
- Akiyama and Wang (1981)
- M Ribeiro Filho, J. T. Pinho : Grid based + Delaunay algorithm (2001)

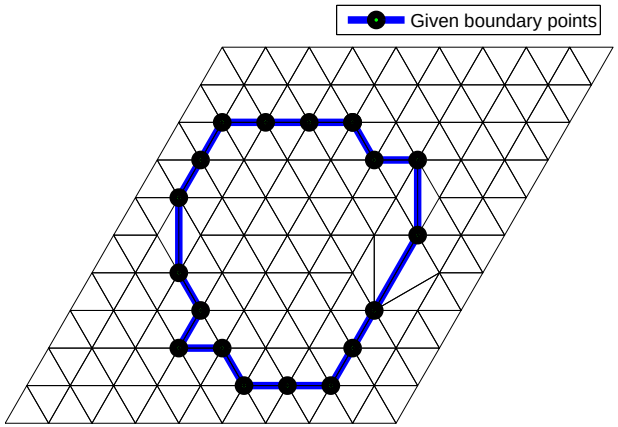
Our algorithm



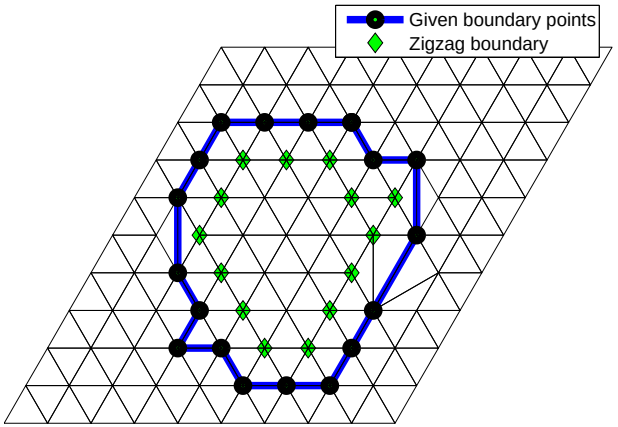
(a)



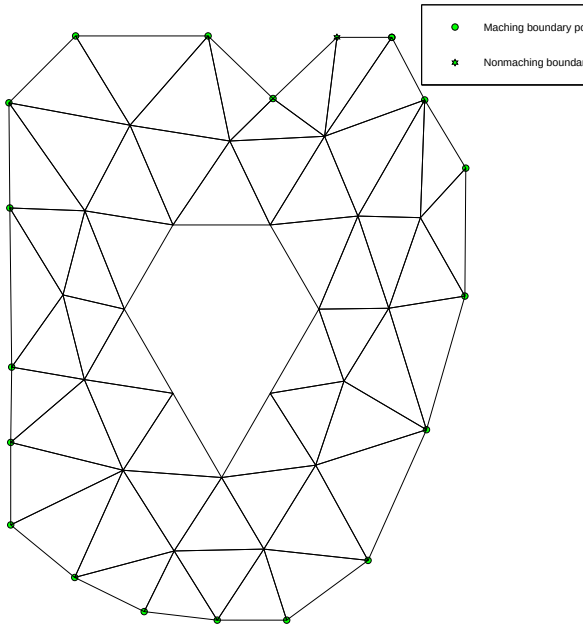
(b)


Given boundary points

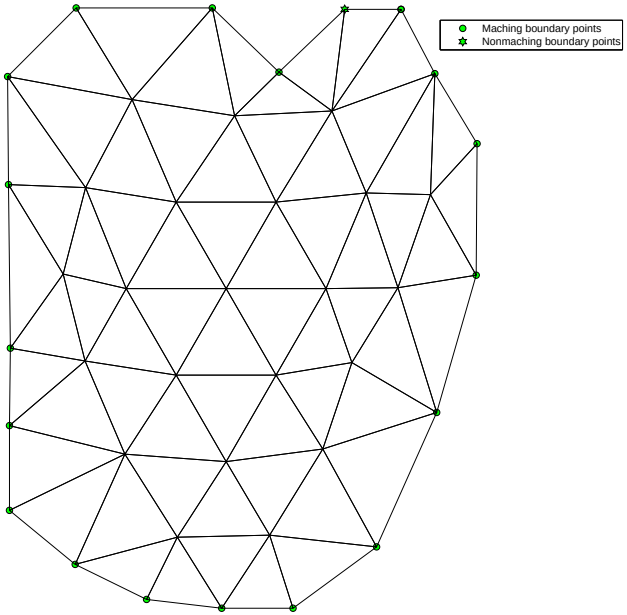
(a)


Given boundary points
Zigzag boundary

(b)

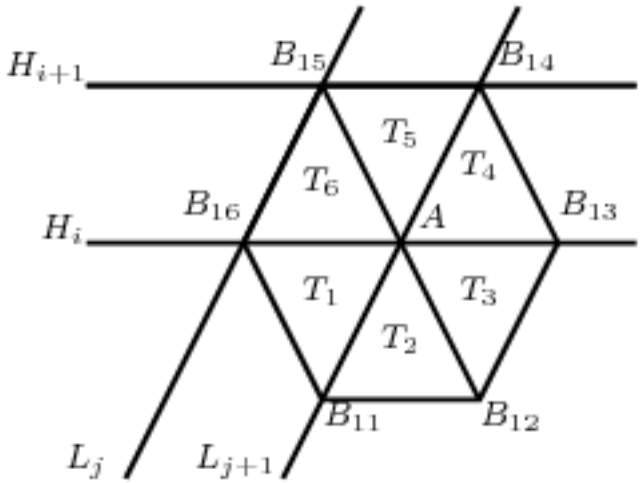

Maching boundary p
Nonmaching bounda

(a)


Maching boundary points
Nonmaching boundary points

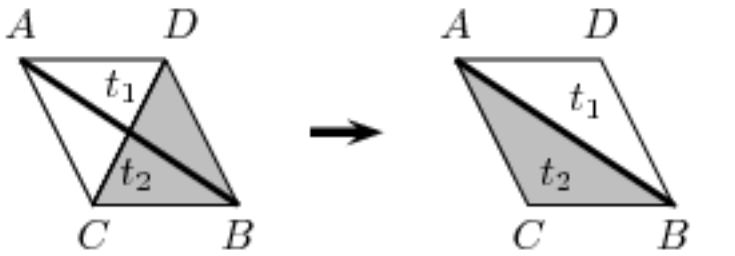
(b)

Rules of numbering of grid points and cells



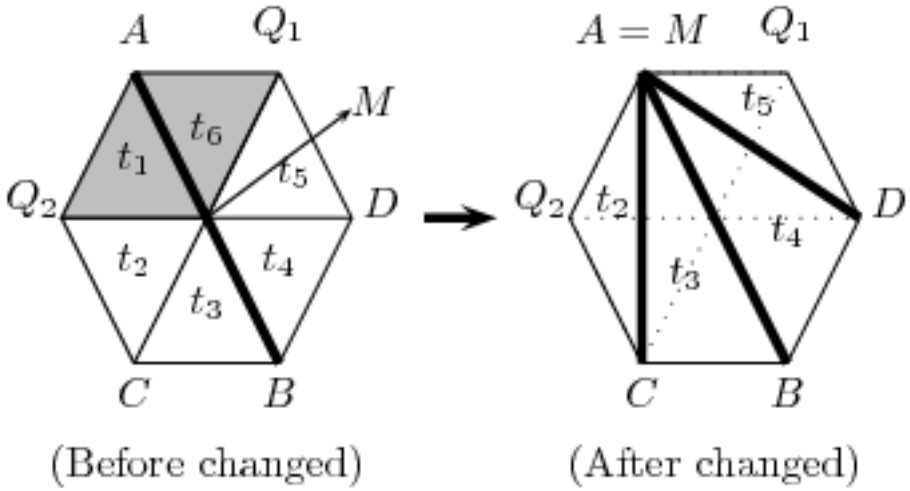
(a) $\mathtt{id}(A) = (i-1)\mathtt{lx} + j + 1$; $\mathtt{id}(T_6) = 2(i-1)(\mathtt{lx} - 1) + 2j - 1$; $\mathtt{id}(T_5) = 2(i-1)(\mathtt{lx} - 1) + 2j$

Operations



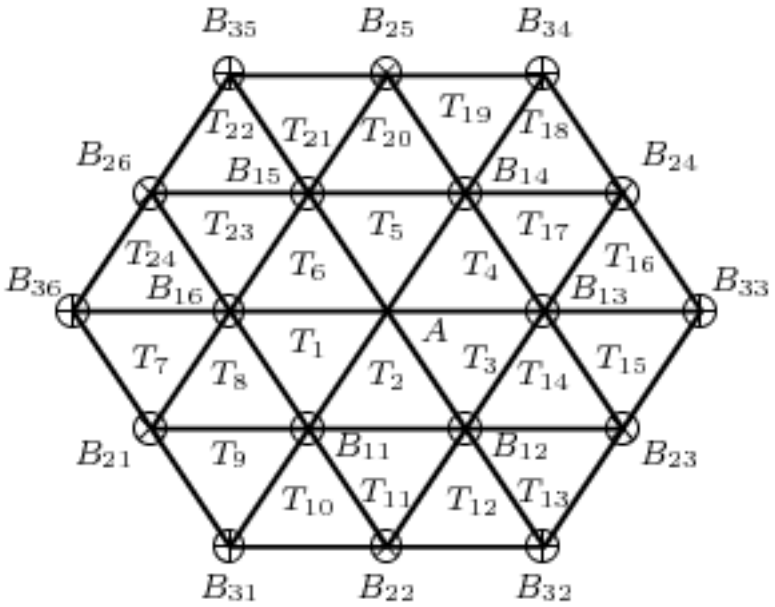
(Before changed) (After changed)

(a) E_2-type: Change two grid cells $\triangle ADC$, $\triangle CBD$ as $\triangle ADB$, $\triangle CBA$



(b) E_3-type: Delete two grid cells $\triangle AQ_1M$, $\triangle AQ_2M$ and then push M into A

Local Grid Template



(b) Three cases of location of B in $T_{24}(A)$; E_1 type case: $\odot$, B_{1i}; E_2 type case: $\otimes$, B_{2i}; E_3 type case: $\oplus$, B_{3i}

Characteristic matrix

$$CM = \begin{pmatrix}
-1x & -1x+1 & 1 & 1x & 1x-1 & -1 \\
-1x+1 & 1 & 1x & 1x-1 & -1 & -1x \\
-1 & -1x & -1x+1 & 1 & 1x & 1x-1 \\
-1x-1 & -21x+1 & -1x+2 & 1x+1 & 21x-1 & 1x-2 \\
-1x & -1 & 1x-1 & 1x & 1 & -1x+1 \\
-21x+2 & -21x+3 & -21x+4 & 1 & 0 & -1 \\
-21x+1 & -41x+6 & -21x+5 & 2 & 21x-3 & -2 \\
-21x+1 & -1x+2 & 1x+1 & 21x-1 & 1x-2 & -1x-1 \\
-21x+3 & -21x+4 & 1 & 0 & -1 & -21x+2 \\
-41x+5 & -41x+8 & 3 & 21x-2 & 21x-5 & -21x \\
-41x+6 & -21x+5 & 2 & 21x-3 & -2 & -21x+1
\end{pmatrix}$$