GEOMETRIC MODELS FOR ACTIVE CONTOURS

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ABSTRACT

A geometric formulation of active contours for 2D, 3D boundary detection and motion tracking is presented. The technique is based on active contours evolving in time according to intrinsic geometric measures of the image. The evolving contours naturally split and merge, allowing the simultaneous detection of several objects and both interior and exterior boundaries. The proposed approach is based on the relation between active contours and the computation of minimal distance curves or minimal surfaces in a Riemannian space whose metric is derived from the image. Previous models of geometric active contours are improved, allowing stable boundary detection when their gradients suffer from large variations, including gaps. Numerical experiments are also presented.

1. INTRODUCTION

Since original work by Kass *et al.* [1, 2], extensive research was done on "snakes" or active contour models for boundary detection. The classical approach is based on deforming an initial contour $\mathcal{C}_0$ towards the boundary of the object to be detected. The deformation is obtained by trying to minimize a functional designed so that its (local) minimum is obtained at the boundary of the object. This energy model is not capable of handling changes in the topology of the evolving contour when direct implementations are performed, and special, many times heuristic, topology handling procedures must be used [3]. The approach is also non-intrinsic.

Recently, novel geometric models of active contours were simultaneously proposed by Caselles *et al.* [4] and by Malladi *et al.* [5]. These models are based on the theory of curve evolution and geometric flows, which has received a large amount of attention from the image analysis community in recent years [6, 7, 8, 9]. The approach allows automatic changes in the topology when implemented using the level-sets based numerical algorithm [10].

In [11], we studied the relation between both models for two dimensional object detection, proposing what we called "geodesic active contours." We first proved that, for a particular case, the classical energy approach is equivalent to finding a geodesic curve in a Riemannian space with a metric derived from the image (see also [12]). This means that the boundary we are looking for is the path of minimal distance, measured in the Riemannian metric, that connects two given image points. We then showed that assuming a level set representation of the deforming contour, we can find this geodesic curve via a geometric flow which is very similar to the one obtained in the geometric approaches mentioned above. However, this geodesic flow includes a new component in the curve velocity, based on image information, which improves those models. In [13] we extended the "geodesic active contour" model to three dimensional object detection. We showed that the desired boundary is given also by a surface of minimal area, where the area is defined in a non-Euclidean space. We shall review both models below with some detail. Finally, combining this circle of ideas with the geometric active contour model for motion tracking proposed in [14] we propose the corresponding "geodesic active contour model for motion tracking". As in the previous geometric models, this model will be able to track several moving objects in a sequence of images and resolve occlusions without special handling procedures. In all of these cases one can show that the solution to the geodesic flow exists in the viscosity framework, is unique and verifies stability estimates. We shall illustrate the results obtained with these models in a series of examples of real and synthetic images.

2. GEODESIC ACTIVE CONTOURS

2.1. The geodesic curve flow

Let us briefly describe the classical energy based snakes. Let $\mathcal{C}(q) : [0,1] \to \mathbf{R}^2$ be a parametrized planar curve and let $I : [0,a] \times [0,b] \to \mathbf{R}^+$ be a given image in which we want to detect the objects boundaries. The classical snakes approach [1] associates the

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curve $\mathcal{C}$ with an energy given by

$$E(\mathcal{C}) = \alpha \int_0^1 |\mathcal{C}'(q)|^2 dq + \beta \int_0^1 |\mathcal{C}''(q)|^2 dq - \lambda \int_0^1 |\nabla I(\mathcal{C}(q))| dq, \quad (1)$$

where $\alpha, \beta, \lambda \in \mathbf{R}^+$. The first two terms control the smoothness of the contours to be detected (internal energy), while the third term is responsible for attracting the contour towards the object in the image (external energy). Solving the problem of snakes amounts to finding, for a given set of constants α, β, and λ, the curve $\mathcal{C}$ that minimizes E. Note that the approach is not topology independent, when considering more than one object in the image, special topology-handling procedures must be added.

Let us consider a particular class of snakes model where the rigidity coefficient is set to zero, that is, $\beta = 0$. The main reasons motivating this selection were discussed in [11]. Assuming this, and replacing the edge detector $|\nabla I|$ for a general function $g(|\nabla I|)^2$ of the gradient such that $g(r) \to 0$ as $r \to \infty$, we obtain,

$$E(\mathcal{C}) = \alpha \int_0^1 |\mathcal{C}'(q)|^2 dq + \lambda \int_0^1 g(|\nabla I(\mathcal{C}(q))|)^2 dq. \quad (2)$$

The geodesic active contour model will be derived from (2). The functional in (2) is not intrinsic. Actually, if we define a new parametrization of the curve via and substitute in (2), we see that the energies can change in an arbitrary form [11]. To eliminate this degree of freedom we incorporate a constraint. For that, let us define $\mathcal{U}(\mathcal{C}) := -\lambda g(|\nabla I(\mathcal{C})|)^2$, and write $\alpha = m/2$. Therefore, $E(\mathcal{C}) = \int_0^1 \mathcal{L}(\mathcal{C}(q))dq$, where $\mathcal{L}$ is the Lagrangian given by $\mathcal{L}(\mathcal{C}) := \frac{m}{2}|\mathcal{C}'|^2 - \mathcal{U}(\mathcal{C})$. The Hamiltonian (see [11]) is then given by $H = \frac{p^2}{2m} + \mathcal{U}(\mathcal{C})$, where $p := m\mathcal{C}'$. Minimizing now the energy functional (2) constraining the Hamiltonian to zero energy level, choice which is justified in detail in [11] and which amounts to fix as zero the energy level of an ideal edge, we get by means of Maupertuis' principle that minimizing $E(\mathcal{C})$ given by (2) submitted to the constraint $H = 0$ is equivalent to minimizing

$$Min \int_0^1 g(|\nabla I(\mathcal{C}(q))|)|\mathcal{C}'(q)| dq. \quad (3)$$

Thus, when trying to detect an object, we are not just interested in finding the path of minimal classical length but the one that minimizes a new length definition which takes into account image characteristics.

In order to minimize (3) we search for the gradient descent direction of (3). Thus, according to the steepest-descent method, to deform the initial curve $\mathcal{C}(0) = \mathcal{C}_0$ towards a (local) minima of (3), we should follow the curve evolution equation

$$\frac{\partial \mathcal{C}(t)}{\partial t} = g(I)\, \kappa \vec{\mathcal{N}} - (\nabla g \cdot \vec{\mathcal{N}})\vec{\mathcal{N}}, \quad (4)$$

where κ is the Euclidean curvature, $\vec{\mathcal{N}}$ is the unit inward normal, and the right hand side of the equation is given by the Euler-Lagrange of (3) as derived in Appendix B in [11].

2.2. The level-sets geodesic flow

The geodesic flow (4) is represented using the level-sets approach [10], which we proceed to describe.

Assume that the curve $\mathcal{C}$ is a level-set of a function $u : [0, a] \times [0, b] \to \mathbf{R}$. That is, $\mathcal{C}$ coincides with the set of points u = constant (e.g. $u = 0$). This representation is parameter free, then intrinsic. It is easy to show, see for example Appendix C in [11] and [10], that if the planar curve $\mathcal{C}$ evolves according to $\mathcal{C}_t = \beta \vec{\mathcal{N}}$, for a given function β, then the embedding function u should deform according to $u_t = \beta|\nabla u|$, where β is computed on the level-sets. By embedding the evolution of $\mathcal{C}$ in that of u, topological changes of $\mathcal{C}(t)$ are handled automatically and accuracy and stability are achieved using the proper numerical algorithm [10]. Therefore, we obtain that solving the geodesic problem is equivalent to searching for the steady state solution ($\frac{\partial u}{\partial t} = 0$) of the following evolution equation ($u(0, \mathcal{C}) = u_0(\mathcal{C})$):

$$\frac{\partial u}{\partial t} = |\nabla u| \operatorname{div}\left(g(I)\frac{\nabla u}{|\nabla u|}\right) \quad (5)$$

That means, (5) is obtained by embedding (4) into u with $\beta = g(I)\kappa - \nabla g \cdot \vec{\mathcal{N}}$. On the equation above we made use of the fact that $\kappa = \operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right)$. Equation (5) is the main part of the proposed active contour model.

3. THE MINIMAL SURFACE MODEL

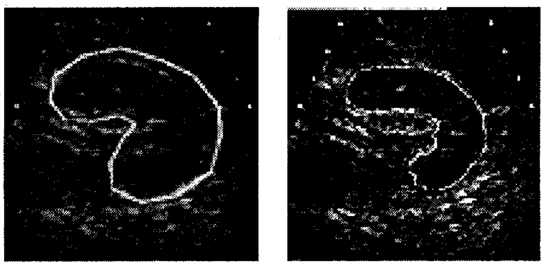
We may extend the previous model to object detection in 3D images. In the case of surfaces the "weighted" length is replaced by a "weighted" area

$$A_R := \int\int g(I)da, \quad (6)$$

where da is the (Euclidean) element of area.

Taking a level set representation, in analogy with (5), the steepest descent method to minimize (6) gives

$$\frac{\partial u}{\partial t} = |\nabla u| \operatorname{div}\left(g(I)\frac{\nabla u}{|\nabla u|}\right) \quad (7)$$



(a) Original image (b) Final contour
Fig.1. Foetus detection in ultrasound image

However, now u is a function of (t, x) whose level sets at fixed t are generically surfaces. Comparing Equations (5),(7) with previous geometric models [4, 5] we see that the term $\nabla g \cdot \nabla u$, naturally incorporated via the geodesic framework, is missing in them. This term attracts the curve to the boundaries of the objects (∇g points toward the middle of the boundaries) being of special help when this boundary has high variations, including gaps, on its gradient values. Note that in the old model, the curve stops when $g = 0$. This only happens at an ideal edge. This makes the previous geometric models [4, 5] inappropriate for the detection of boundaries with (un-known) high variations of the gradients. This is in part corrected by the new gradient term.

As in the 2D case, we can add a constant force ([15]), obtaining the general *minimal surfaces model* for object detection:

$$\frac{\partial u}{\partial t} = |\nabla u| \mathrm{div}\left(g(I)\frac{\nabla u}{|\nabla u|}\right) + \nu g(I)|\nabla u|. \quad (8)$$

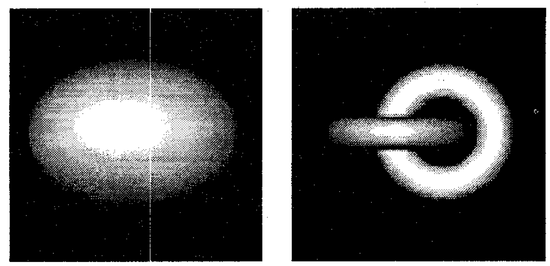
It has the same properties and geometric characteristics as the geodesic active contours, leading to accurate numerical implementations and topology free object segmentation.

4. SNAKES IN MOVEMENT

One can also use these ideas to follow the successive positions of the boundaries of moving objects in a sequence of images $I(t, x)$, where t represents the continuous time. The idea is to add a term to the equation (8) which gives the displacement of the objects from an image to the next. Hence the resulting model is [14]

$$\begin{aligned} \frac{\partial u}{\partial t} &= |\nabla u| \mathrm{div}\left(g(t,x)\frac{\nabla u}{|\nabla u|}\right) + \nu g(t,x)|\nabla u| \\ &+ (1 - g(t,x))v \cdot \nabla u \qquad (9) \end{aligned}$$

where v is the velocity vector field and $g(t, x)$ is a smooth function such that $g(t, x) = 1$ if $|\nabla \hat{I}(t, x)| \leq k - \epsilon$ and $g(t, x) = 0$ if $|\nabla \hat{I}(t, x)| \geq k + \epsilon$ where k is a given threshold and ϵ is supposed to be small and $\hat{I}$ denotes a smoothed version of I. The first and second terms of the rigth hand side above and the initial condition have the same interpretation as in model (5). The second term tries to keep the estimated contour or snake near the boundary of the moving object (see [14]). An experiment will be presented in next section.



(a) Initial surface (b) Detected surface
Fig.2. Detection of two linked torus

5. EXPERIMENTAL RESULTS

Let us present some examples of the proposed geometric active contours model (5),(7). The numerical implementation is based on the algorithm for curve evolution via level-sets developed by Osher and Sethian [10] and recently used by many authors for different problems in computer vision and image processing (see the references). In the following figures, we follow the level set $u = 0$.

Figures 1(a),(b) show the detection of a foetus in an ultrasound image. Figure 1(a) is the original image with the initial active contour and Figure 1(b) represents the detected contour according to the 2D version of (8). The original image was smoothed with a Gaussian kernel before the detection was performed. This avoids some local minima, and together with the attraction force provided by the new term, allowed to detect an object with gaps in its boundary. Figure 2 is a test example showing the possibility of detecting some kind of linked or knotted surface starting from an ellipsoid containing it. Figure 2(a) represents the initial surface which contains inside of it two linked torus like in Figure 2(b). Figure 2(b) represents the detected surfaces according to (8). We can observe the change of topology during the evolution. Figure 3 shows a sequence of images where we want to follow the contour of the ball using the equation (9). Figure 3(a) shows the initial image with the initial active contour inside of the ball. Figure 3(b) shows the contour found by the 2D version of the geometric active contour model (8). Figures 3(c) and 3(d) show the contour found by evolving the contour according to (9).

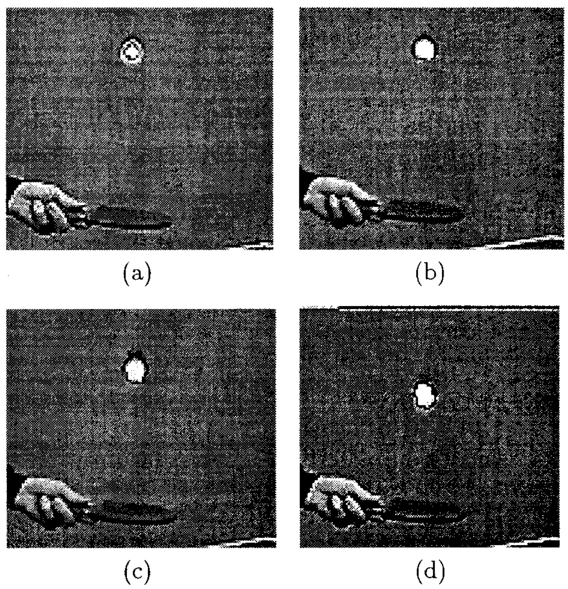


Fig.3. Tracking of a moving ball

6. CONCLUDING REMARKS

We presented a geometric formulation of active contour models for 2D, 3D boundary detection and motion tracking. They were based in the level set formulation of an evolution equation trying to minimize a weighted length or area which interprets edges or object boundaries as geodesic curves or minimal surfaces with respect to a Riemannian metric derived from the image. We adapted this model to be able to track one or several objects in an image sequence. As mentioned above, the geometric formulation enables us to find one or several objects simultaneously without special topology handling procedures. Moreover, as shown in [4, 11, 13] one can prove existence, uniqueness and stability results for those models and check its correctness at least for ideal step edges. We also presented numerical experiments in real and synthetic images to show its performance.

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References

[1] M. Kass, A. Witkin, and D. Terzopoulos, "Snakes: Active contour models," *IJCV* **1**, pp. 321-331, 1988.

[2] D. Terzopoulos, A. Witkin, and M. Kass, "Constraints on deformable models: Recovering 3D shape and nonrigid motions," *AI* **36**, pp. 91-123, 1988.

[3] R. Szeliski, D. Tonnesen, and D. Terzopoulos, "Modeling surfaces of arbitrary topology with dynamic particles," *Proc. CVPR*, pp. 82-87, 1993.

[4] V. Caselles, F. Catte, T. Coll, F. Dibos, "A geometric model for active contours," *Numerische Mathematik* **66**, pp. 1-31, 1993.

[5] R. Malladi, J. A. Sethian and B. C. Vemuri, "Shape modeling with front propagation: A level set approach," *IEEE-PAMI* **17**, pp. 158-175, 1995.

[6] G. Sapiro and A. Tannenbaum, "Affine invariant scale-space," *IJCV* **11:1**, pp. 25-44, 1993.

[7] B. B. Kimia, A. Tannenbaum, and S. W. Zucker, "Shapes, shocks, and deformations, I," *IJCV*, to appear.

[8] R. Kimmel, A. Amir, A. M. Bruckstein, "Finding shortest paths on surfaces using level sets propagation," *IEEE-PAMI*, to appear.

[9] L. Alvarez, F. Guichard, P. L. Lions, and J. M. Morel, "Axioms and fundamental equations of image processing," *Arch. Rational Mechanics* **123**, 1993.

[10] S. J. Osher and J. A. Sethian, "Fronts propagation with curvature dependent speed: Algorithms based on Hamilton-Jacobi formulations," *J.of Comp. Physics* **79**, pp. 12-49, 1988.

[11] V. Caselles, R. Kimmel, and G. Sapiro, "Geodesic avtive contours," submitted, September 1994.

[12] S. Kichenassamy, A. Kumar, P. Olver, A. Tannenbaum and A. Yezzi, "Gradient flows and geometric active contour models," *Proc. ICCV*, Cambridge MA, June 1995.

[13] V. Caselles, R. Kimmel, G. Sapiro and C. Sbert, "Three Dimensional Object Segmentation via Minimal Surfaces and Geometric Deformable Models," March 1995, submitted.

[14] V. Caselles, B. Coll, "Snakes in Movement," *SIAM J. on Numerical Analysis*, to appear.

[15] L. D. Cohen, "On active contour models and balloons," *CVGIP* **53**, pp. 211-218, 1991.

[16] F. Leymarie and M. D. Levine, "Tracking deformable objects in the plane using an active contour model" *IEEE-PAMI* **16**, pp. 617-634, 1993.