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Point Pattern Matching Algorithm Using Unit-Circle Parametrization

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ABSTRACT

This paper presents only a matching algorithm based on Delaunay triangulation and Parametrization from the extracted minutiae points. This method maps local neighborhood of points of two different point sets to unit-circle using topology information by Delaunay triangulation method from feature points of real fingerprint. Then, a linked convex polygon that includes an interior point is constructed as one-ring which is mapped to unit-circle using Parametrization that keep shape preserve. In local matching, each area of polygon in unit-circle is compared. If the difference of two areas are within tolerance, two polygons are consider to be matched and then translation, rotation and scaling factors for global matching are calculated.

Key words: Point pattern matching, Feature based matching, Fingerprint matching, Parametrization.

1. INTRODUCTION

As one of the variants in pattern recognition, point pattern matching is to match 2 point sets and consequently identify if they are identical. Input of point pattern matching are 2 point sets, however, it could also be images that could be derived into point sets. Generally, point patterns are defined by 2-d point sets, of which each point could be represented by Euclidean coordinates (x_i, y_i), and the point sets are represented as $P = p_i, i = 1, 2, ..., n$. Where n is the number of points.

Pattern recognition provides solution to many problems in real life such as in biometric system, personal identification of banks etc. It matches two point sets and consequently identify if they are identical. This is applicable in fingerprint recognition with minutiae as a representation, which has been widely used as an individual identification method. Fingerprint is still a commonly used method in personal identification these days. Point pattern matching is one of the studies in the development of fingerprint recognition. Computation can be made less expensive by reducing the storage. An efficient way to represent fingerprints is in the form of minutiae [1,2].

The fingerprint represented by minutiae can be presented as feature points also. By using the feature points, fingerprint matching problem can be solved using point pattern matching. The scaling differences, noise and unrecorded part of the fingerprints contribute distortions therefore it is necessary to have a distortion-tolerant point pattern matching algorithm. Many of such algorithms have been developed in recent years, such as [1,3,4,5].

This paper presents only a matching algorithm based on Delaunay triangulation and Parametrization from the extracted feature points. This method maps local neighborhood points of two different point sets to unit-circle using topology information

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Fig. 1. Example of point pattern from fingerprints.

by Delaunay triangulation method from feature points. Then, a linked convex polygon that includes an interior point is constructed as one-ring which is mapped to unit-circle using Parametrization that keeps shape preserve.

In this paper, rotation factor is calculated using one-ring neighbor of two mapped unit-circle. First, we calculate the one-ring neighbor polygon area based on boundary points. If the difference of two polygon's area is less than a threshold value, we assume the two one-ring neighbors are matched. Finally, rotation factor can be calculated. Then rotation factor is used to calculate the difference of two polygons angle. The difference of the translation factor can be calculated by moving center point of raw data.

The content of this paper is as follows. In section 2, the Wamelen's approach which is famous recently is explained. In section 3, our proposed algorithm is explained followed by the detail processing steps. The conclusion and future works are presented in section 4.

2. RECENT ADVANCEMENT IN POINT PATTERN MATCHING

Recently there are researches on the point pattern matching by comparing two point sets, measuring the differences of these 2 point sets and hence matching the points. Points are matched if they are within acceptable threshold [1,4]. These methods measure the level of similarity of the points and their nearest neighbors in term of transformation between the 2 point sets. In Van Wamelen's paper [1], an algorithm with combination of measurement of sorted nearest neighbors and probability technique was presented to be one that is able to match the point pattern with fast speed and low computation cost.

The algorithm consists of three main parts: the pre-computation, local matching and global matching. In pre-computation stage, nearest neighbors lists and matching distance is defined. The matching distance is used during the local matching of point sets, in order to check if there exists a point within this distance on set 1 after a specific transformation on a point from set 2. In local matching, small subsets are extracted from point sets, so called the local-neighborhood, and the percentage of similarity is checked. For each point in subset, we have to check, if there a counterpart point exists in the compare set, if the number of points matched is within the acceptable percentage, a local match is said to be achieved. In order to extract the subset local match, obtaining a list of neighbors to be measured. Global matching is being executed on the output list of local matching. Global matching applies the least-square estimation method to measure the level of similarity between the output lists of local match, while the local matching criteria are based on the Euclidean coordinates and threshold defined in the pre-computation, the global matching produce a transformation function by least-square estimation based on the result from local matching. The transformation function that consists of scaling, rotation and translation:

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \end{pmatrix} + s\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \quad (1)$$

where t_x, t_y are the translation factor, θ the rotation factor, and s the scaling factor.

$\begin{pmatrix} x \\ y \end{pmatrix}$ belongs to a point in set P and $T\begin{pmatrix} x \\ y \end{pmatrix}$ would be the translated point in Q should the point is transformed by the factor.

In local matching, point set P is said to be matched with Q if there exists ρ number of elements that fits this criteria:

$$|T_{s,\theta,t_x,t_y}(p) - q| < t, \quad t = \lambda \frac{r}{(2\sqrt{n})} \quad (2)$$

Two parameters define the precision of matching, the matching probability, $\rho \in [0,1]$ and the matching size, $t \in R^{+}$. Pre-computation defines the allowable threshold. The method first takes random point in P and then search for a point in Q such that it matches the point in P locally, if such similarity transformation exists then, global matching will be tested. It utilizes the concept that which part of the point set P would have higher probability of matching to the corresponding point set Q. This observation enables the idea of local matching, in which a number of points in Q maintains topology or structure that closely resembles to those corresponding points in P, if they are from the same source, even though distorted by translation, scaling, or rotation can exist in other parts [1].

3. THE PROPOSED MATCHING METHOD USING UNIT CIRCLE

Our method of unit circle mapping applies in similar manner through the following procedures:

1) Obtaining feature points.
2) Delaunay triangulation of the feature points.
3) Mapping local triangulation neighborhood to unit-circle.
4) Matching mapped unit-circle.

Similarly, dividing the procedure into 3 main parts, we study a method to find the rotation factor in local matching with triangulation of point pattern and parameterized circle unit.

3.1 Triangulation of point set

The triangulation can be constructed by Delaunay triangulation. Triangulation of point pattern provides a way of defining neighborhood and thus omits the necessity of finding k-nearest neighbors. Delaunay triangulation is known as the dual graph of Voronoi diagram. In next step, we will map one-ring neighbor to unit-circle using the topology information that be constructed by triangulation method. The result of general triangulation is shown in Figure 2.

In this paper, we applied triangulation method using G. Leach's algorithm [6].

We can construct topology information of real fingerprint feature points using G. Leach's

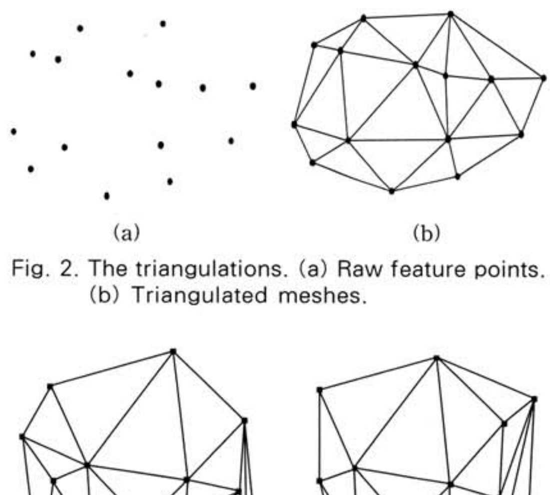


Fig. 2. The triangulations. (a) Raw feature points. (b) Triangulated meshes.

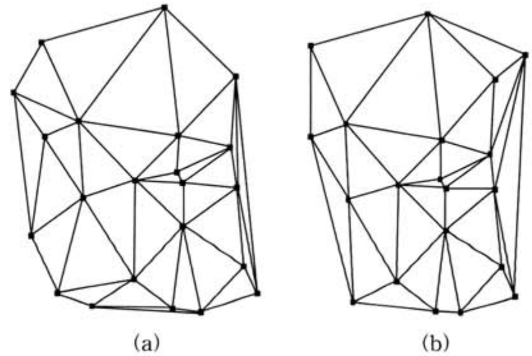


Fig. 3. Triangulation of point sets from real fingerprint feature points.

algorithm. The result of triangulation with real fingerprint feature points is shown in Figure 3.

3.2 Parametrization of triangulation

Introduced in M.S. Floater's Parametrization and smooth approximation of surface triangulations, the Parametrization of surface triangulation to R^2 domain was done by piece-wise linear system that solves the interior points by barycentric combination of local neighborhoods and chord length mapping of boundary points [7]. A method of choosing the weight as the average of barycentric coordinate of a geodesic polar mapping local sub-triangulation neighborhood was also being introduced as shape-preserving. It first maps local neighborhood of a point from R^3 into a R^2 plane by geodesic polar mapping. New position of the point is the barycentric weighted coordinate of its neighbors, defined as the averaged barycentric combination of series of sub-triangles by a definition [7]. In local matching problem, we map a local neighborhood to unit circle by chord length of local boundary points. For 1-ring neighborhood, the geodesic polar mapping could be omitted since the point set model is in R^2.

3.2.1 Parametrization of triangulation

Boundary points are the information that we need in finding the interior point in the method of finding the interior points. The boundary points can be found with chord length calculation. In this method, we have to select one point from the triangulation. We shall map the selected point to a boundary location in unit circle. This boundary location is decided by us and it could be changed depending on the application of the Parametrization.

Let b be the boundary edge on triangulations. We calculate boundary point location on the circle by:

$$dA_i = \frac{b_i}{\sum_i^{n_b} b_i} \times 360 \qquad (3)$$

where n_b is a number of boundary points. The method calculates the ratio of each boundary edge to the sum of all boundary edges, and applies the ratio to the 360 degree and apply this angle, dA, to place the point on the circumference of the circle. So, we can allocate the boundary points on the circumference as coordinate:

$$x = r \times \cos(dA)$$
$$y = r \times \sin(dA)$$
$$r: \text{radius of the} \circ \leq .$$

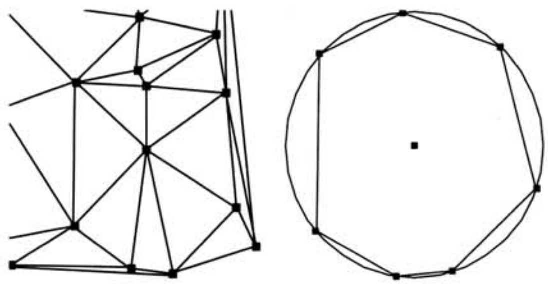


Fig. 4. Boundary points mapping in unit circle.

3.2.2 Barycentric coordinate for center point

In the calculation of weights for the shape-preserving parametrization, an average of barycentric method was introduced. Let points p_1, p_2, p_3 be the 3 vertices of the sub-triangulation of an local triangulation neighborhood of center point p. The weight for the barycentric coordinate of p here denoted as λ_{i,j_k}, is defined as:

$$\lambda_{i,j_k} = \frac{area(p,p_2,p_3)}{area(p_1,p_2,p_3)},$$

$$\lambda_{i,j_k} = \frac{area(p,p_1,p_3)}{area(p_1,p_2,p_3)},$$

$$\lambda_{i,j_k} = \frac{area(p,p_1,p_2)}{area(p_1,p_2,p_3)}$$

A local triangulation neighborhood of 1-ring, edge, and face neighborhood usually consists of more than 3 points. Floater's method defines a rule of forming sub-triangles from the local triangulation neighborhood, and measure the barycentric coordinate of p as the average barycencer of all sub-triangles in this neighborhood. For each $l \in 1,2,\ldots,d_i$ extend a straight line from p_l through center point and let the line intersects the local neighborhood polygon at a second point. This point would be either a vertex $p_{r(l)}$ or a location on the boundary edge of the local triangulation formed by $p_{r(l)}$ and $p_{r(l)+1}$.

The result is $r(l) \in 1,2,\ldots,d_i$ and δ_1, δ_2, δ_3 such that $\delta_1 > 0$, $\delta_2 > 0$, $\delta_3 > 0$, $\delta_1 + \delta_2 + \delta_3 = 1$ and $p = \delta_1 p_l + \delta_2 p_{r(l)} + \delta_3 p_{r(l)+1}$ [7].

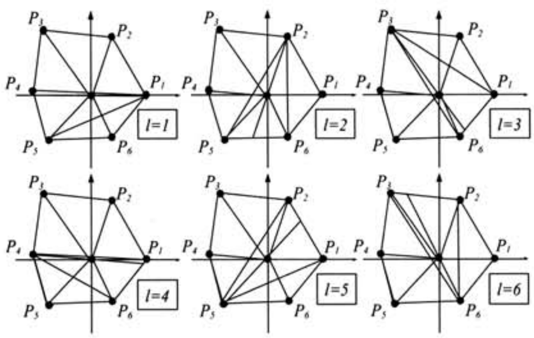


Fig. 5. from l=1,...,di and k=1,..., di.

The weight, δ, in each barycentric sub-triangle was defined as such:

$$\mu_{k,l} \text{ for } k=1,\ldots,d_i,\ \mu_{l,l}=\delta_1,\ \mu_{r(l),l}=\delta_2,\ \mu_{r(l)+1,l}=\delta_3, \quad \text{otherwise, } \mu_{k,l}=0$$

In which the barycentric of each sub-triangle is:

$$p = \sum_{k=l}^{d_i} \mu_{k,l} p_k, \quad \sum_{k=l}^{d_i} \mu_{k,l} = 1, \quad \mu_{k,l} \geq 0$$

As a result, the weight λ_{i,j_k} for each local neighbor is:

$$\lambda_{i,j_k} = \frac{1}{d_i} \sum_{l=1}^{d_i} \mu_{k,l}, \quad k=1,\ldots,d_i \tag{4}$$

We use these weights as the barycentric weights of the center point of the 1-ring neighborhood. Resulting is an average barycenter of the local neighborhood from point set P and Q, namely c_p and c_q. It is possible to use the angle differences between c_p and c_q as a guide for the amount of rotation required for local matching.

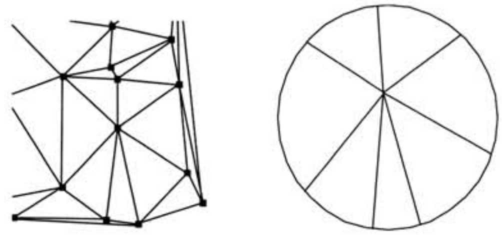


Fig. 6. Unit circle mapping.

3.3 Matching algorithm

In this paper, our proposed algorithm is shown in Figure 7.

The first step is precomputation, with 2 input data sets P and Q. Precomputation is done in two methods: triangulation of P and Q, and unit circle

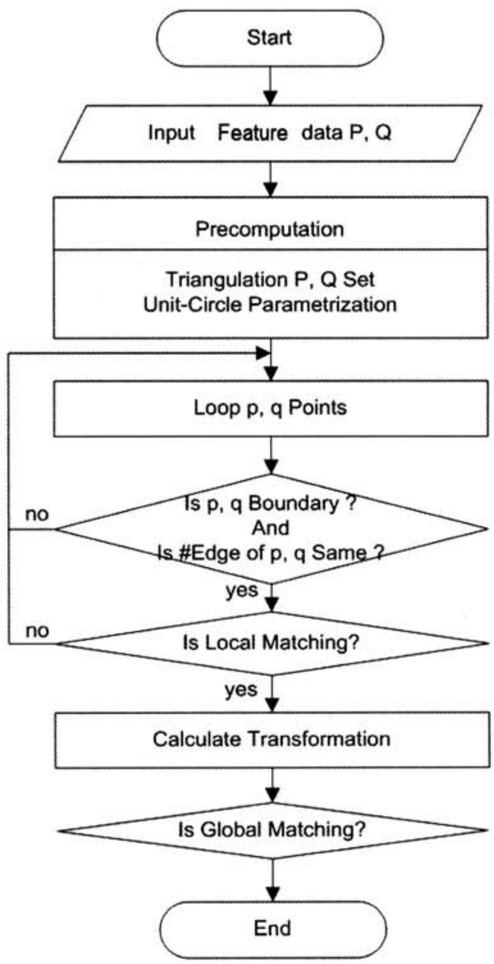


Fig. 7. Flowchart of the proposed point pattern matching algorithm.

parametrization. In second step, we perform local matching. p and q are control points in data set P and Q respectively. We need to check on all the combinations of p and q to see if both are interior points and the number of edge of both points are same. If both statements are true, we continue with the local matching. Else, we proceed to the next p, q combination. In the following step of local matching, we produce one-ring neighbor polygon for p and q, and the difference of the area is compared. If the difference is less than a pre-defined threshold value, then we said that both p and q are locally matched. We then calculate the transformation factor.

After finishing the checking of all the p, q combination and obtaining the transformation factor, we perform global matching. In global matching, we try to overlap P and Q together based on the transformation factor. If the number of points matching is higher than the matching factor rho, we conclude that both data sets are matched. The time is recorded at this moment as well.

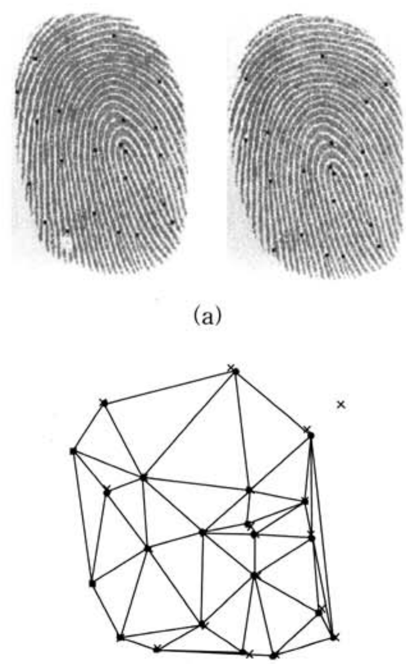
Local matching algorithm matches the local neighborhood of two triangulated point sets. We already calculated the one-ring neighbor polygon using Shape-preserving Parametrization method in previous section, and calculated the one-ring neighbor polygon area based on boundary points. If the difference of two polygon's area is less than threshold value, we assume the two one-ring neighbors are locally matched. Then rotation factor is used to calculate the difference of two polygons angle. The difference of the translation factor can be calculated by moving center point of raw data. The local matching is not affected by the rotation of the polygon because what we are considering is not the orientation but the area if the area of the polygon's compared is same that means the polygons are matched.

4. IMPLEMENTATION

The algorithm described above was implemented in the VC++ (MFC) programming language on a 32-bit processor Intel 1.66GHz. We tested the program on many randomly generated data sets. A pair of rotated unit circle mapped from a pair of pre-defined rotated model with same topology, proved to be same. Experimenting with real data was done with a pair of fingerprint as seen in figure 1. The distortion affected the topology of point pattern models which consequently affects the circle unit. The similarity between local neighborhoods also created difficulty in matching.

4.1 Case Study

To test our algorithm on a real world situation, we applied it to a fingerprint recognition problem. We took two fingerprint image from The FVC2002 fingerprint database (see Figure 8). In Figure 8 (b) we give a representation of the match found. The over wrapped points represent the point set Q.



(a)

(b)

Fig. 8. (a) Two different fingerprints from the same finger with the feature points. (b) The match our implementation found between the two sets of fingerprint feature point.

4.2 Random point sets

We generated random point sets by using Wamelen's method, in order to test our algorithm.

Table 1 and Table 2 show the result of Wamelen's algorithm and our proposed algorithm respectively. By comparing the result in both tables, the Wamelen's algorithm takes 1.4113 seconds in average when n=2m case, but our proposed algorithm takes 0.0022 seconds in average when the same case. so our proposed algorithm much faster than Wamelen's it. Besides the tables show us same result in other cases.

Table 1. The result of Wamelen's implementation (Time)

Parameter				Average time in second for		
m	rho	eta	lambda	n=2m	n=4m	n=8m
50	0.95	0.175	0.428	0.02	0.053	0.089
100	0.95	0.175	0.428	0.051	0.099	0.23
200	0.95	0.175	0.428	0.13	0.257	0.518
400	0.95	0.175	0.428	0.347	0.704	1.401
800	0.95	0.175	0.428	0.759	1.596	3.802
50	0.6	0.25	0.61	0.504	1.065	2.852
100	0.6	0.25	0.61	1.208	2.102	4.479
200	0.6	0.25	0.61	2.854	6.612	11.853
400	0.6	0.25	0.61	8.257	17.007	34.475
50	0.9	0.4	0.84	0.138	0.322	0.613
100	0.9	0.4	0.84	0.282	0.607	1.614
200	0.9	0.4	0.84	0.797	1.495	3.61
400	0.9	0.4	0.84	3.001	5.862	11.491

Table 2. The result of our algorithm implementation (Time)

Parameter				Average time in second for		
m	rho	eta	lambda	n=2m	n=4m	n=8m
50	0.95	0.175	0.428	0.00062	0.00116	0.00202
100	0.95	0.175	0.428	0.00117	0.00216	0.00347
200	0.95	0.175	0.428	0.0024	0.00346	0.00618
400	0.95	0.175	0.428	0.00367	0.00611	0.01209
800	0.95	0.175	0.428	0.00639	0.0121	0.02381
50	0.6	0.25	0.61	0.00058	0.00121	0.00219
100	0.6	0.25	0.61	0.00122	0.00218	0.00356
200	0.6	0.25	0.61	0.00194	0.00346	0.00626
400	0.6	0.25	0.61	0.00356	0.00636	0.01211
50	0.9	0.4	0.84	0.0006	0.00115	0.00214
100	0.9	0.4	0.84	0.0012	0.00216	0.00332
200	0.9	0.4	0.84	0.00218	0.00349	0.00648
400	0.9	0.4	0.84	0.00326	0.00652	0.01207

5. CONCLUSIONS

In this paper we studied point pattern matching, Parametrization of triangulation and the application of the Parametrization in point pattern matching. Wamelen V. introduced a method of point pattern matching by comparing sorted-nearest neighbors. The method sort the nearest neighbors of each point in the two point sets, P and Q, and matches them by comparing whether there is a point which exists in the counterpart with the calculated translation factor differences.

In the local matching method we proposed, we adapted the weight calculation method from shape-preserving Parametrization. We calculate weights the unit circle enter point from the convex combination of their point pattern local neighborhood.

The matching algorithm for point pattern introduced in this paper is fast and efficient. In the proposed approach there is no need to calculate translation, rotation and scaling changes in point patterns. Therefore it avoids huge calculation in comparison to earlier methods [1,4]. Due to this reason the approach has a reduced complexity and is easy to implement in practical systems. For future enhancement, we would like to further optimize our algorithm and try to achieve higher matching rate.

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