z-Transform and Convolution

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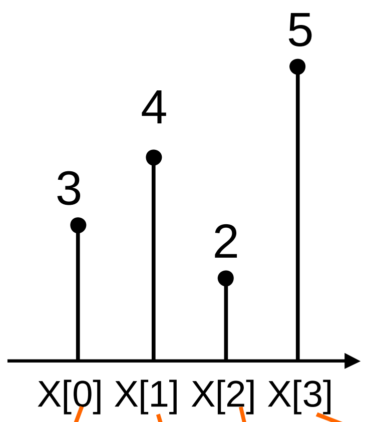
Z-Transform

Discrete Time Signal <u>**x**</u>



$$\underline{\mathbf{X}} = \{3, 4, 2, 5\} = \begin{bmatrix} 3 \\ 4 \\ 2 \\ 5 \end{bmatrix}$$

$$X(z) = 3*z^{0} + 4*z^{-1} + 2*z^{-2} + 5*z^{-3}$$

A Polynomial

$$X(z) = \sum_{n=0}^{N-1} x[n] z^{-n}$$

Z-Transform

Z-Transform

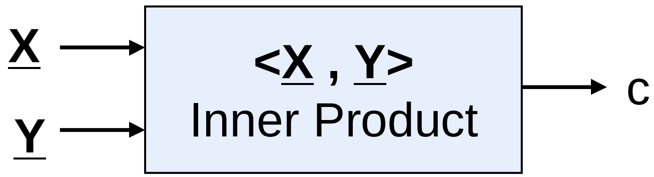
$\underline{X} = \{1, 4, 2, 5, 0, 5\}$ → Z-Transform → $X(z) = 1{*}z^{0} + 4{*}z^{-1} + 2{*}z^{-2} + 5{*}z^{-3} + 5{*}z^{-5}$

$X(z) = 4{*}z^{-1} + 5{*}z^{-5}$ → Inverse Z-Transform → $\underline{X} = \{0, 4, 0, 0, 0, 5\}$

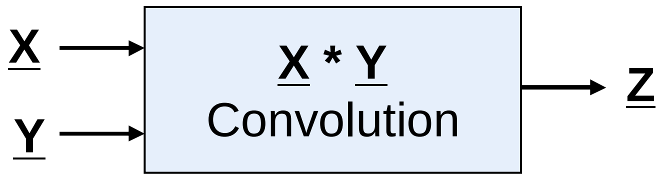
$\underline{X} = \{0, 0, 0, 0, 0, 1\}$ → Z-Transform → $X(z) = ?$

$X(z) = z^{-3}(4 + 5{*}z^{1} + 6{*}z^{2})$ → Inverse Z-Transform → $\underline{X} = ?$

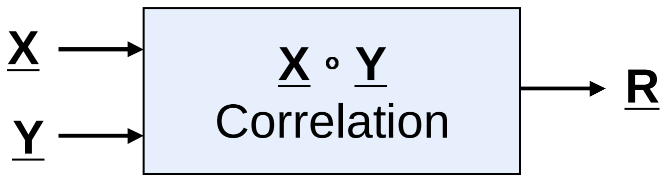
Operations in Discrete Time Signals



$$\langle \underline{X}, \underline{Y} \rangle = \sum_{i}^{N} x[i]y[i]$$

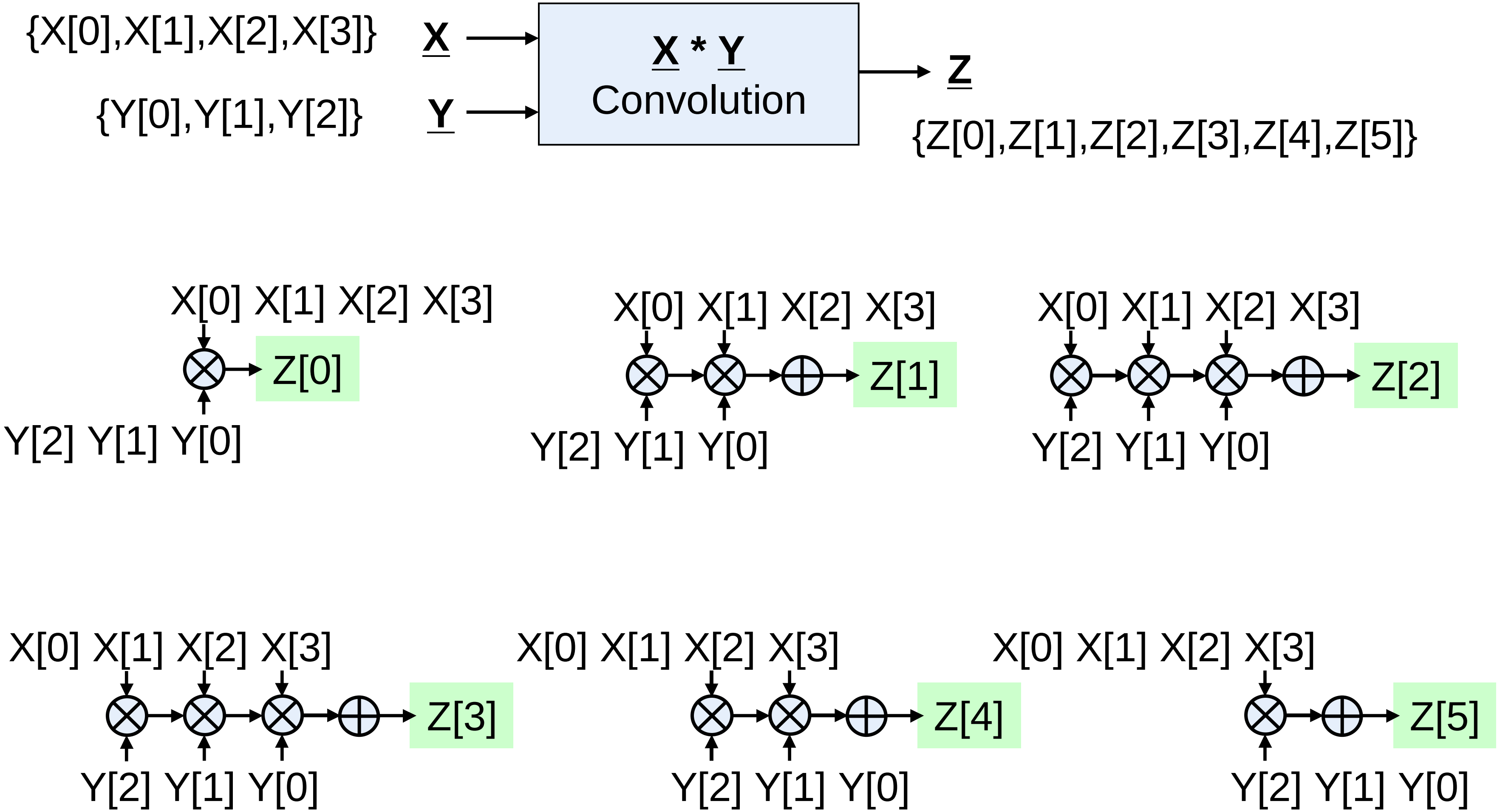


$$Z[n] = \sum_{i} x[i]y[n-i]$$

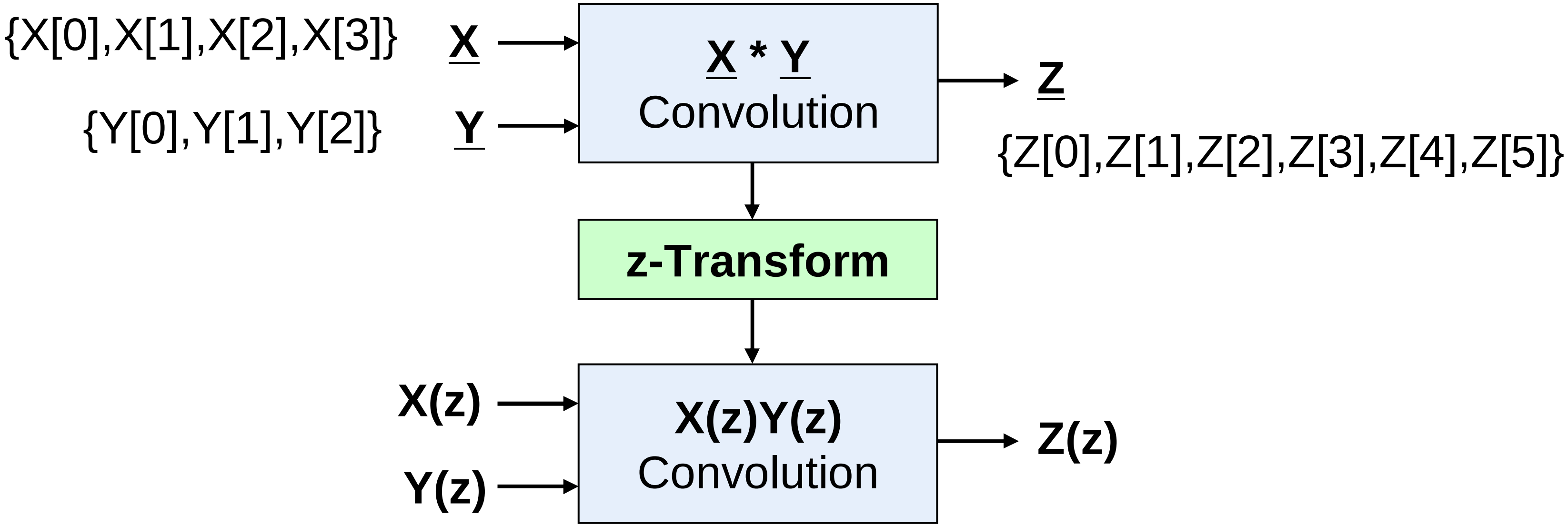


$$R[n] = \sum_{i} x[i]y[i+n]$$

Convolution in Discrete Time Signals



Convolution in z-Transform Domain



$X(z) = x[0]+x[1]z^{-1}+x[2]z^{-2}+x[3]z^{-3}$

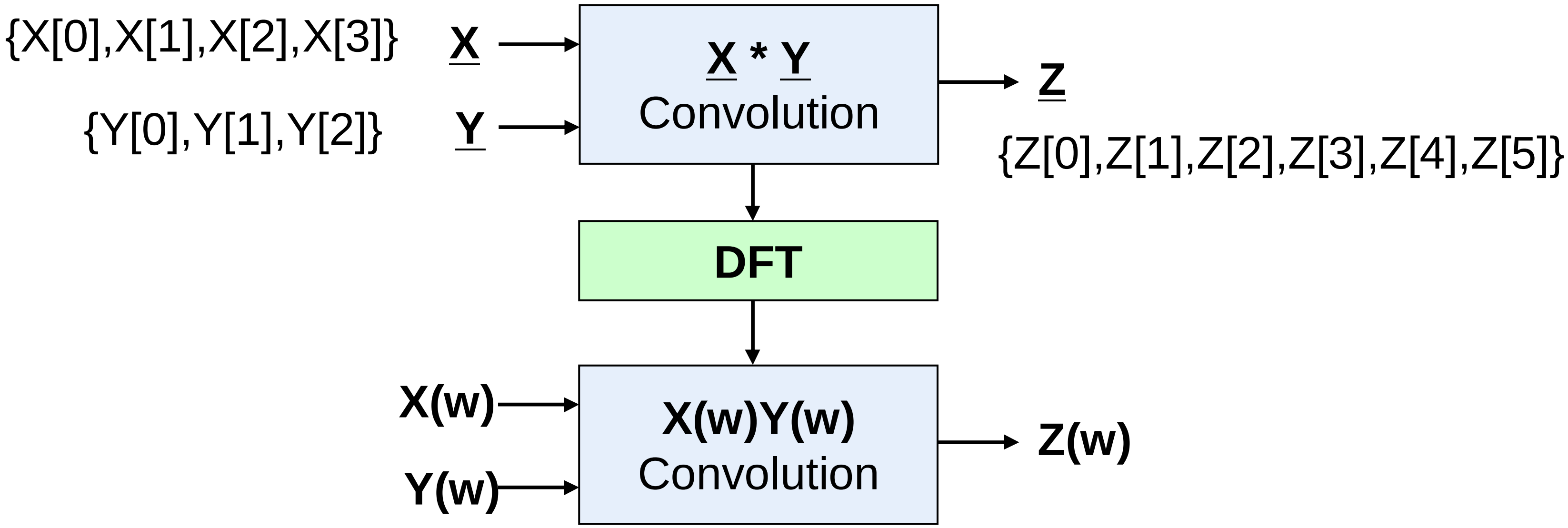
$Y(z) = y[0]+y[1]z^{-1}+y[2]z^{-2}$

$Z(z) = (x[0]+x[1]z^{-1}+x[2]z^{-2}+x[3]z^{-3})(y[0]+y[1]z^{-1}+y[2]z^{-2})$

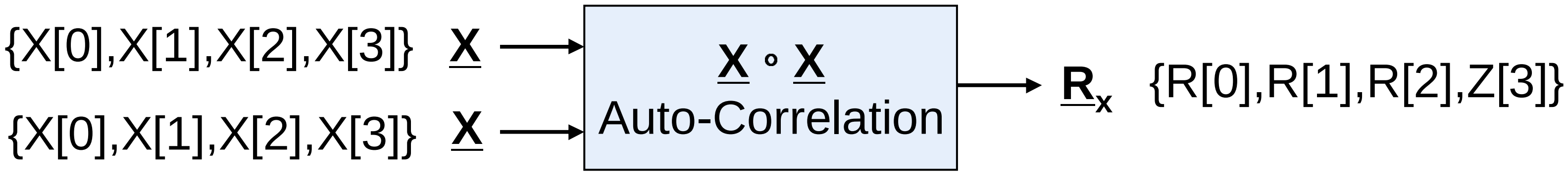
$Z[0] = x[0]y[0]$ $Z[1] = x[0]y[1]+x[1]y[0]$ $Z[2] = x[0]y[2]+x[1]y[1]+x[2]y[0]$

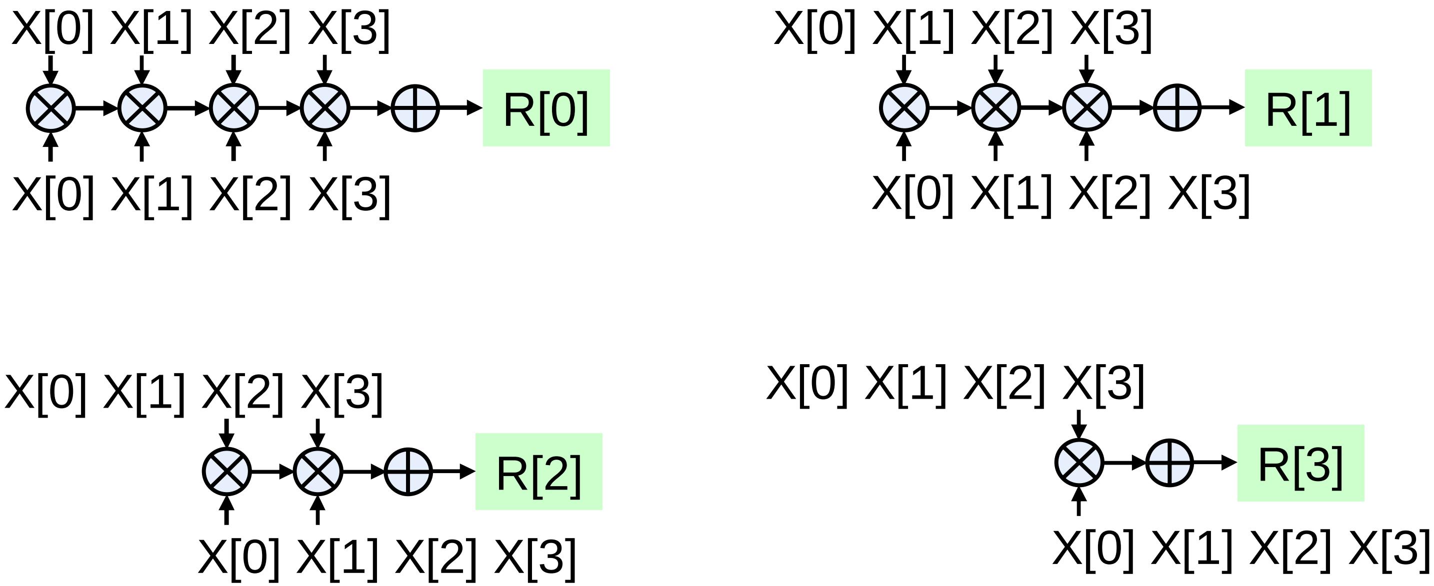
$Z[3] = x[1]y[2]+x[2]y[1]+x[3]y[0]$ $Z[4] = x[2]y[2]+x[3]y[1]$ $Z[5] = x[3]y[2]$

Convolution in the DFT Domain



Correlation in Discrete Time Signals





Summary

- Z-Transform
- Convolution
- Correlation